Blending Kinematic and Software Models for Tighter Reachability Analysis

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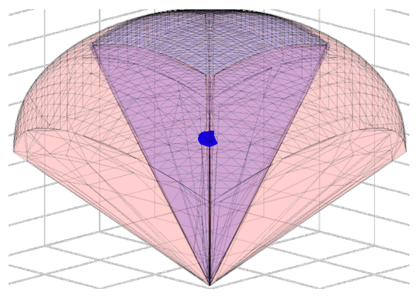
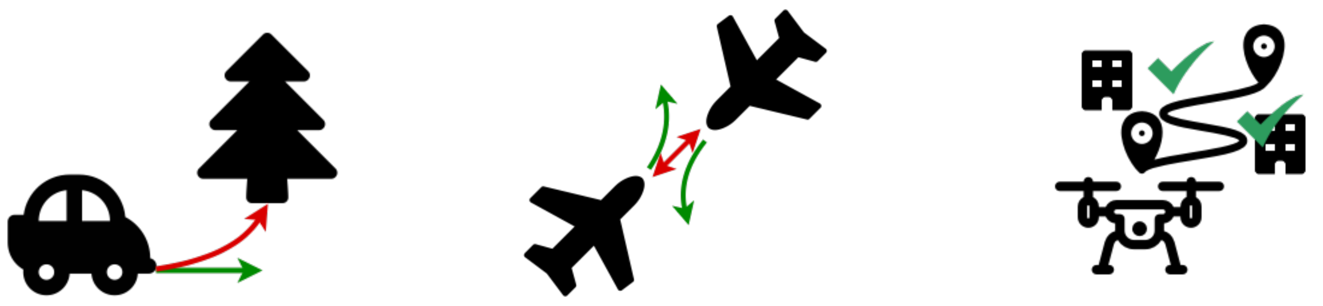


Problem

Computing a **reachable sets** is at the center of many challenging tasks for mobile autonomous systems:

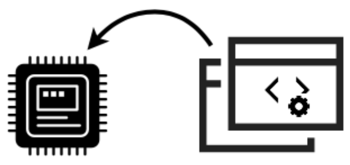


Obstacle Avoidance — Aircraft collision avoidance — System safety and liveness

Currently the **calculation of reachable sets** is parametrized exclusively with the **system's physical attributes**.

This ignores the fact that these systems are **driven by sophisticated software components** that **juxtapose another set of constraints on the system**.

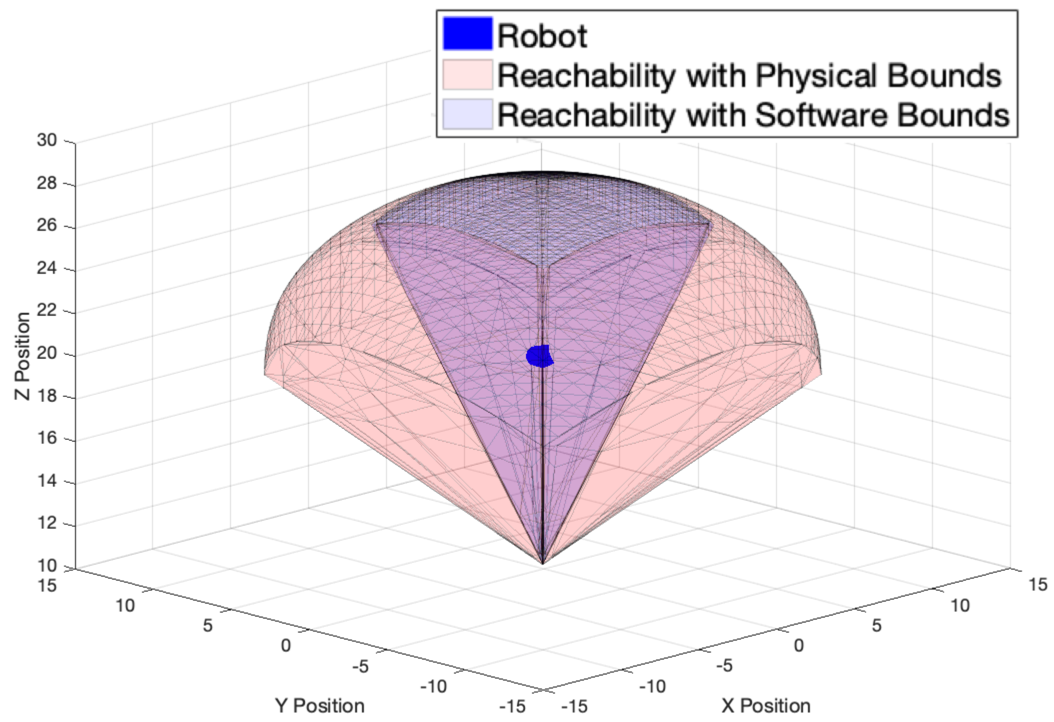
Constraints



Hardware — Software

Reachable Sets

A reachable set is the area or volume a robot can reach in a given amount of time.

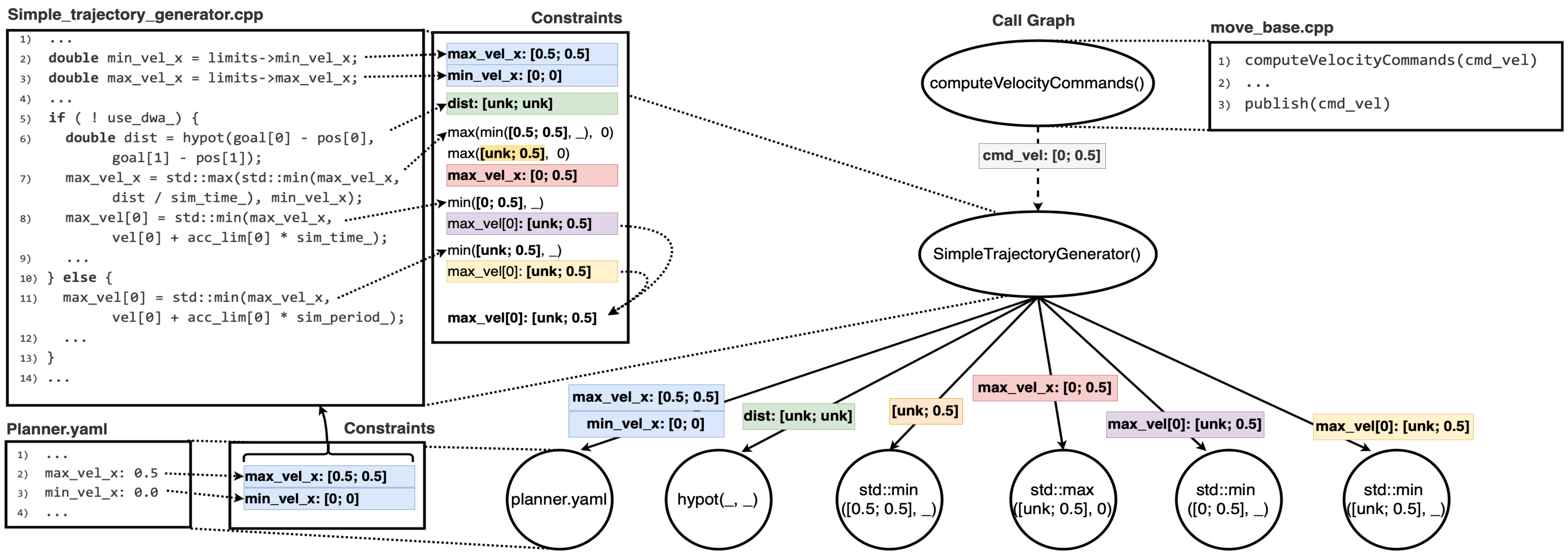


For example, the reachable set for the Elre quadrotor is shown above.

Proposed Solution

Insight: the precision of a reachable set could be dramatically higher by considering the constraints imposed by software.

The approach **finds constraints on program variables** which **control the physical behavior of the robot**. For example, the variable *cmd_vel* controls the robots physical velocity and is shown to be bounded between [0, 0.5] in software.



Preliminary Results

The exploratory study was run on the Elre quadrotor and the Husky robot. We found reductions in reachable sets of up to 91%.

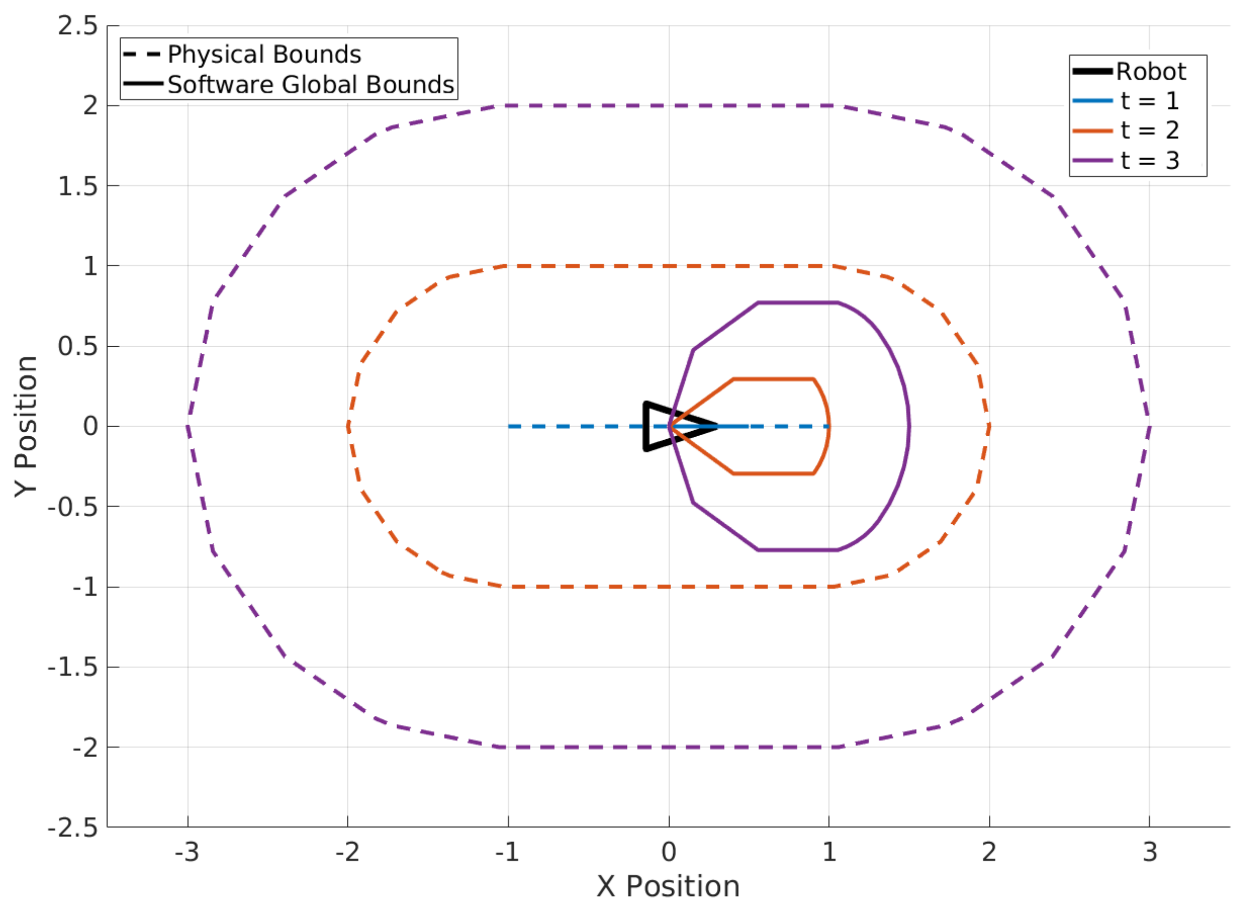
We found software bounds for **5 out of 6** program variables.

Robot Type	Physical Bounds	Software Bounds
Husky (Differential Drive)	Max Velocity: 1 m/s Min Velocity: -1 m/s Turn Rate: 2 rad/s	Max Velocity : 0.5 m/s Min Velocity: 0 m/s Turn Rate: 0.63 rad/s
Erle Quadrotor	Thrust: 45 N Max Pitch: 45 $degrees$ Max Roll: 45 $degrees$	Thrust: ? N Max Pitch: 19 $degrees$ Max Roll: 19 $degrees$

Using software bounds we see **reductions in reachable sets by up to 91%.**

Robot type	Physically Bound Reachability	Software Bound Reachability	Reduction
Differential Drive ($t = 3s$)	$20.24m^2$	Max Velocity: $17.10m^2$ Min Velocity: $15.10m^2$ Velocity: $3.77m^2$ Max Turn Rate: $17.06m^2$ All Constraints: $1.85m^2$	16% 25% 81% 16% 91%
Quadrotor ($t = 3s$)	$716930m^3$	Max Pitch: $343428m^3$ Max Roll: $343428m^3$ All Constraints: $163563m^3$	52% 52% 77%

Over time the the physical bounds reachable set grows faster than the set computed with the software bound. For example, **the Husky's software bound reachability at t = 3s is a subset of the physically bounded reachable set at t = 2s.**



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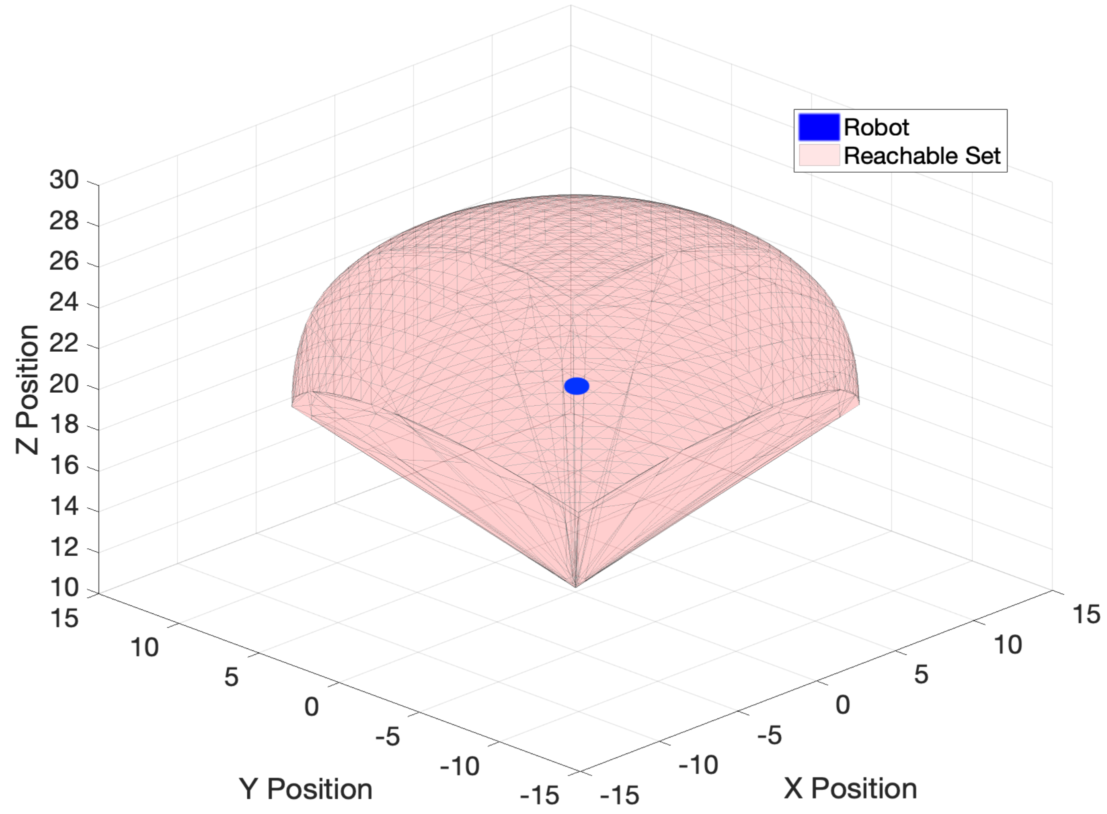
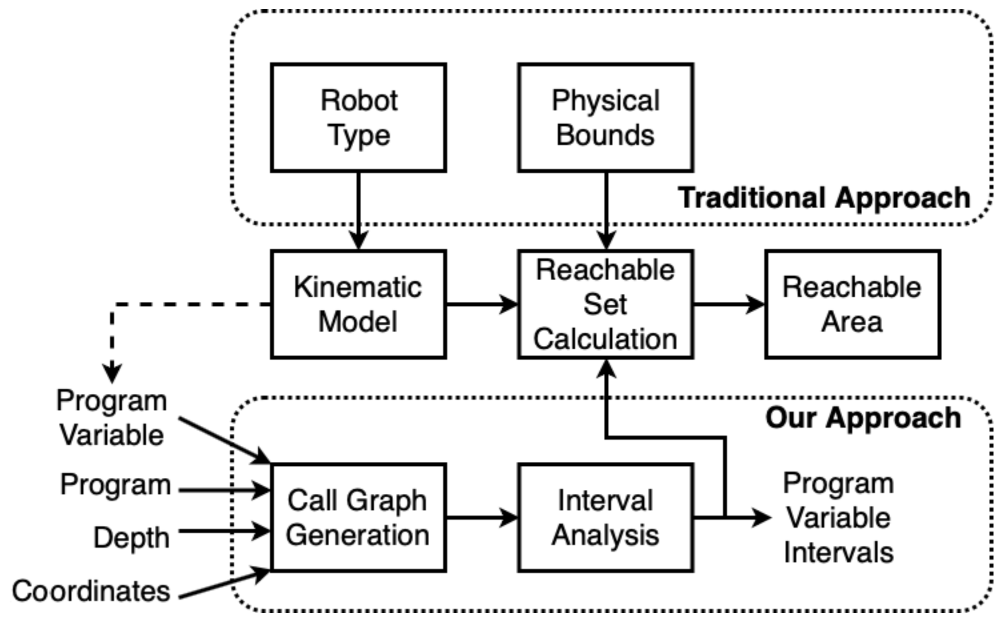
Unused figures





Algorithm 1: find_bounds

```
Input: v, x, cur_depth
bounds = [NaN, NaN];
x.visited = True;
if cur_depth < d then
    cur_depth ++ ;
    for node in x.callers and x.callees do
        if node.visited == false then
            b = find_bounds(v, node, cur_depth);
            bounds = bounds ∪ b
        end
    end
end
final_bounds = interval_analysis(x, bounds);
return final_bounds(v) ;
```

[1] Dabit Industries. 2019. Erle-Copter drone kit. https://dabit.industries/products/ erle- copter- drone- kit.
[2] IEEE. 2019. Robots - Husky. https://robots.ieee.org/robots/husky/.